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# MIGRATION OF EXPLOSION BUBBLES IN A ROTATING TEST TANK

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MIGRATION OF EXPLOSION BUBBLES IN A
ROTATING TEST TANK

By:

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ABSTRACT:  The upward migration of pulsating explosion bubbles under the
influence of buoyancy (gravity) cannot be studied on a model scale in free
water, but only in test tanks.  If a stationary test tank is employed the
air pressure above the water surface must be reduced to assure similitude.
This report deals with another alternative, namely a tank which is subjected
to a suitably selected acceleration during the explosion test.  The case
considered is that where the acceleration is produced by a centrifuge.

The Coriolis accelerations distort the motions of the migrating bubble in
two ways.  First, the "upward" (i.e. toward the water surface) motion of the
bubble is deflected to the side.  This sideward motion can be observed
experimentally.  In the second place, the rotation affects the magnitude
of the "upward" motion and introduces a systematic error which cannot be
detected by experimental methods.

The bubble migration in a rotating system is treated using the simplifying
assumption that the bubble retains its spherical shape.  The "upward" as
well as sideward migration is calculated for the first and second cycle of
the pulsation.  In a typical example, the systematic error in the "upward"
migration was about 1%, which is probably less than the experimental accuracy
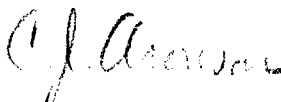of such studies.

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MIGRATION OF EXPLOSION BUBBLES IN A ROTATING TEST TANK

The design of a high gravity tank for the study of underwater explosion phenomena is greatly simplified if the tank is accelerated by a centrifuge instead of a linear accelerator. This report shows that the rotation does not introduce errors which exceed the experimental accuracy of such tests. Therefore, the rotating tank is a promising tool for model experiments on migrating explosion bubbles.

This paper is the outcome of calculations which were made during the planning studies for a rotating high gravity tank. Since the background as well as the details of these calculations are of interest to the users of such tanks, they are made available in this report.

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Captain, USN
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By direction

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## ILLUSTRATIONS

## LIST OF SYMBOLS

a        =        Reduced bubble radius = $A/A_m$

$\tilde{a}_m$        =        Reduced maximum bubble radius of the classical bubble theory

A        =        Bubble radius

$A_M$        =        Maximum bubble radius

$A_m$        =        Minimum bubble radius

B        =        Constant  See (49)

c        =        $A_M^3/A_m^3 - 1$

C        =        Constant in (30)

g        =        Acceleration of gravity

K        =        Integration  constant

q        =        Constant in (47)

P        =        Pressure **(absolute)**

$P_o$        =        Hydrostatic pressure **(absolute)**

$\vec{r}$        =        Radius vector in moving system

$\vec{R}$        =        Radius vector in resting system

$r(\theta)$        =        Radius (Used in polar coordinates $r$, $\theta$)

R        =        Distance of point of explosion from center of rotation

s        =        length of path

T        =        Period of bubble pulsation

$T_1$ = First period

$T_2$ = Second period

$t$ = Time

$\tilde{t}$ = Reduced period of the classical bubble theory

$t_s$ = Time of impact

$v$ = Velocity

$W$ = Charge weight

$X, Y$ = Coordinates in the resting system

$\Delta Z$ = Gravity migration in the resting system

$x, y$ = Coordinates in the rotating system

$\Delta y$ = Sideward migration in the rotating system

$\Sigma \Delta z$ = Total "upward" migration in the rotating system

$\Delta z$ = "Upward" migration in the rotating system caused by centrifugal acceleration

$\Delta z_c$ = Distortion of upward migration by Coriolis acceleration

$\Delta z_z$ = Distortion of upward migration by the variation of centrifugal acceleration with z

Subscripts to migration terms refer

1 to first cycle
2 to second cycle
i to i-th cycle
M to bubble maximum
m to bubble minimum

$Z_0$ = Total hydrostatic head = Depth + head of atmospheric pressure (used in connection with bubble parameters)

| | | |
|---|---|---|
| $\alpha$ | = | $1 + c^{-1}$ |
| $\beta$ | = | See (9) |
| $\theta$ | = | Angle |
| $\eta$ | = | Coordinate normal to axis of symmetry |
| $\rho$ | = | Density |
| $\tau$ | = | $(2t/T) - 1$ = Reduced time |
| $\varphi$ | = | Velocity potential |
| $\phi$ | = | Angle |
| $\psi$ | = | Angle |
| $\Psi$ | = | Stream Function |
| $\Psi^*$ | = | Approximate Stream Function |
| $\omega$ | = | Angular velocity |
| $\Omega$ | = | Potential of body force per unit mass |

# I. INTRODUCTION

Various proposals have been made in the past to study the upward motion of underwater explosion bubbles on a model scale in a tank which is subjected to a high acceleration, 1/ to 4/. By this way a gravitational field can be established which assures similitude of the bubble motions induced by buoyancy. To some extent this objective can be also fulfilled in a "vacuum tank". Reduction of the air pressure above the water can provide the conditions for similitude of bubble migration. This method requires that certain additional conditions are satisfied, namely either a specific water temperature so that a controlled boiling occurs at the surface of the bubble or the use of a liquid of low vapor pressure and special explosives of low density. In both cases it is necessary to rely heavily on theory in order to specify the test conditions 5/. Such a study can hardly be considered as a true experiment, but rather as a combination of a theoretical and experimental approach. Nevertheless, quite satisfactory results have been obtained with this method 5/ and 6/.

Since shockwave and bubble pulse phenomena are not scaled by the vacuum tank technique, this method fails for phenomena where pressure effects as well as bubble migration are important. Examples are: effect of the migrating bubble on targets, interaction of the bubble with the water surface, and surface phenomena. (Studies of the latter effects will require the use of special media in the tank, for instance the use of a layer of dust at the water surface separated from the water by a membrane. This would also make possible the study of base surge phenomena.) In a vacuum tank it is also impossible to simulate the migration and condensation phenomena of steam bubbles such as are produced by nuclear underwater explosions. For all these purposes a high gravity tank is needed, as discussed in 7/.

The high gravity tanks originally proposed employed linear accelerators. Subsequently, R. S. Price (NOL) has proposed consideration of a tank mounted on a centrifuge and pointed out the great advantages in structural design, ease, and safety of operation of this arrangement and the practically unlimited duration available for the explosion test. Tests yielding encouraging results have been made with a small centrifuge at the Naval Ordnance Laboratory 9/ and a large centrifuge at Sandia Corporation, New Mexico. Evaluation of the latter tests showed satisfactory agreement with the bubble migration theory presented in 7/. Although this theory reproduces all existing experimental evidence with good accuracy, this comparison is hardly sufficient to demonstrate that bubble migration can be

properly simulated in a rotating tank. In particular, it is not obvious that motions resulting from the Coriolis accelerations could not adversely affect the migration in such a tank and, thus, introduce a systematic error which, on the basis of an empirical comparison, would remain undetected. It will be shown below that this error is small and that Price's argument is correct that all detrimental effects caused by the rotation decrease with increasing length of the arm on which the test tank is mounted.

## II. EQUATIONS OF MOTION IN A ROTATING SYSTEM

We assume a system rotating about a vertical axis with constant speed and consider the motions in the plane of the rotation only. There are two coordinate systems of interest to us: the stationary coordinate system X,Y and the rotating one x,y. (The rotating coordinate system is that which appears in the photographic pictures of the rotating tank taken with a camera fixed to the rotating system.) The X- and x- coordinates measure the distance from the center of the rotation, the positive direction being away from the center. The Y- and y- coordinates are perpendicular to X and x, positively directed in the direction of rotation.

Although the following equations can be given in the most concise form by means of vectors, we will write the equations also in Cartesian coordinates. The coordinates in the moving system are given in terms of those of the resting system by

$$x = X \cos\omega t + Y \sin\omega t$$

$$y = Y \cos\omega t - X \sin\omega t$$

(1)

or

$$\vec{r} = \vec{R} ,$$

the velocity

$$\dot{x} = \dot{X} \cos\omega t + \dot{Y} \sin\omega t + y\omega$$

$$\dot{y} = \dot{Y} \cos\omega t - \dot{X} \sin\omega t - x\omega$$

or

(2)

$$\frac{d\vec{r}}{dt} = \frac{d\vec{R}}{dt} - \left[\vec{\omega}\,\vec{r}\right] ,$$

the acceleration

$$x^{\cdot\cdot} = X^{\cdot\cdot} \cos\omega t + Y^{\cdot\cdot} \sin\omega t + x\omega^2 + 2 y^{\cdot}\omega$$

$$y^{\cdot\cdot} = Y^{\cdot\cdot} \cos\omega t - X^{\cdot\cdot} \sin\omega t + y\omega^2 - 2x^{\cdot}\omega$$

or                                                                                              (3)

$$\frac{d^2\vec{r}}{dt^2} = \frac{d^2\vec{R}}{dt^2} - \left[\vec{\omega}\left[\vec{\omega}\vec{r}\right]\right] - \left[2\vec{\omega}\,\frac{d\vec{r}}{dt}\right].$$

The angular velocity is denoted by $\omega$. (The vector $\vec{\omega}$ is perpendicular to the x-y and X-Y plane, but the vectors $[\vec{\omega}\vec{r}]$ and $\left[\vec{\omega}\left[\vec{\omega}\vec{r}\right]\right]$ lie within this plane, Figure 1. According to the rules of the vector analysis, the x-components of $[\vec{\omega}\vec{r}]$ and $\left[\vec{\omega}\left[\vec{\omega}\vec{r}\right]\right]$ are $-y\omega$ and $-x\omega^2$, respectively; the y-components $x\omega$ and $-y\omega^2$ respectively.)

Equations (1) and (2) reflect the well known fact that position and velocity in the moving system can be obtained by means of a superposition of the motion relative to the resting system and the motion due to rotation. Such a simple superposition is not applicable to acceleration. A body moving relative to the rotating system behaves as if it were subjected not only to the absolute and the centrifugal acceleration but to an additional supplementary acceleration $-2\left[\vec{\omega}\,\frac{d\vec{r}}{dt}\right]$, called the Coriolis acceleration. This supplementary acceleration is of particular interest in our problem.

If there are no forces acting on a body the absolute accelerations $X^{\cdot\cdot}$ and $Y^{\cdot\cdot}$ vanish. A body free to move, exhibits motions within the rotating system, as if it were accelerated by $x^{\cdot\cdot}$ and $y^{\cdot\cdot}$, equation (3), with $X^{\cdot\cdot} = 0$ and $Y^{\cdot\cdot} = 0$. Relative to the resting system this body moves unaccelerated, i.e. with constant or zero velocity.

The equations of motion of a body free to move are in terms of the rotating coordinates

$$x^{\cdot\cdot} - 2y^{\cdot}\omega - x\omega^2 = 0$$

$$y^{\cdot\cdot} + 2x^{\cdot}\omega - y\omega^2 = 0 .$$

(4)

The solution of these differential equations can be immediately found from (1)

$$x = (x_0 + X_0^{\cdot}\, t) \cos\omega t + (y_0 + Y_0^{\cdot}\, t) \sin\omega t$$

$$y = (y_0 + Y_0^{\cdot}\, t) \cos\omega t - (x_0 + X_0^{\cdot}\, t) \sin\omega t .$$

(5)

The subscript o refers to the position and velocity of the body at t = 0. The velocities relative to the rotating system are

$$x_o^{\cdot} = X_o^{\cdot} + \omega y_o$$

$$y_o^{\cdot} = Y_o^{\cdot} - \omega x_o \quad . \tag{6}$$

## III. SPRAY MOTION

Before applying these equations to the bubble migration, we consider the simple example of the motion of the water spray which is thrown into the air by the underwater explosion. In the case of a test tank which rotates with great speed around a vertical axis, the water surface is essentially a section of the surface of a vertical cylinder. The spray particles which are knocked off by the explosion leave the water surface normally with the velocity $-x_o^{\cdot}$, i.e. they move initially toward the center of the rotation. $y_o^{\cdot}$ is zero for the particle. If we ignore air resistance, such a particle moves unaccelerated in the resting system and equations (5) apply. With (6), we obtain

$$x = (x_o - x_o^{\cdot} t) \cos \omega t + x_o \omega t \sin \omega t$$

$$y = x_o \omega t \cos \omega t - (x_o - x_o^{\cdot} t) \sin \omega t \quad . \tag{7}$$

The radial distance of the particle from the center of rotation is

$$\sqrt{x^2 + y^2} = x_o \sqrt{\omega^2 t^2 + (1 - \beta \omega t)^2} \quad , \tag{8}$$

where

$$\beta = \frac{x_o^{\cdot}}{x_o \, \omega} \qquad \beta \geq 0 \quad . \tag{9}$$

The moment when the particle falls back on the water surface is given by the time t for which the radial distance becomes equal to the original radius $x_o$. This time, $t_s$, is

$$\omega \, t_s = \frac{2 \beta}{1 + \beta^2} \quad . \tag{10}$$

The function (10) has a maximum for $\beta = 1$, i.e. the maximum value which $\omega \, t_s$ can attain is unity.

Figure 2 shows the trajectories of a particle as observed from a point which rotates with the tank. (The water surface is drawn under the assumption

of a very wide tank, for instance a cylindrical shell partly filled with water which rotates around its axis.) The figure illustrates the sideward motion due to the Coriolis forces. In a linearly accelerated tank all these trajectories would be straight "up" and would coincide with the x-axis. If the acceleration of such a tank lasted long enough, every particle would fall back to the same point where it originated, even those which are thrown upward with an exceedingly high velocity. In a rotating tank, such a particle hits the opposite water surface.

The spray motion as observed from a fixed point of observation is less complicated. Price (NOL) has made the following calculations during the planning phase of the centrifugal test tank. From the fixed point of observation, the water surface also appears to be a cylindrical surface. The trajectory of the particle is a straight line A B, Figure 3, with the inclination

$$\tan \phi = \frac{x_0 \, \omega}{\dot{x}_0} \, . \tag{11}$$

The time which it takes for the particle to travel from A to B corresponds to the previous $t_s$. It is obtained from $s = v t_s = x_0 \, \omega t_s / \sin \phi$. From Figure 3 it is immediately seen that

$$\frac{s}{\sin \theta_s} = \frac{x_0}{\sin \phi} \tag{12}$$

and we obtain

$$\omega \, t_s = \sin \theta_s \, , \tag{13}$$

or since    $\theta_s = 2 \psi$  (Figure 3) ,

$$\omega \, t_s = 2 \sin \psi \cos \psi = \frac{2 \tan \psi}{1 + \tan^2 \psi} = \frac{2 \beta}{1 + \beta^2} \, . \tag{14}$$

This is the same relationship as obtained from the equations which apply to the rotating system.

## IV. BUBBLE MIGRATION

The theoretical study of the strong migration of explosion bubbles is considerably complicated by the change of shape which the pulsating bubble undergoes when it contracts to and re-expands from its minimum size. (Bubbles

which migrate slightly remain spherical.) A rapidly migrating bubble is almost exactly spherical at its first maximum, but it assumes a kidney-like shape or even that of a torus at the minimum. This change of shape affects the migration of the bubble. Calculations assuming a spherical bubble shape throughout the pulsation have resulted in migrations too large by about 40%. Since it is our purpose to obtain an estimate of the possible deviation of the migration caused by the rotation of the tank such an over-estimate will not be objectionable, if the calculated deviations are small. This would mean that the actual deviations are smaller. No theory exists today which describes migration and bubble shapes in a satisfactory manner, and it would be an exceedingly difficult undertaking to apply such a theory to a rotating system. Assuming a spherical bubble shape, the migration in a rotating tank can be approximately calculated in closed form.

As is to be expected, rotation introduces two additional migration terms which are due to the Coriolis acceleration: One is in the direction of the "gravity" migration, the other is perpendicular to it. The first of these represents the systematic "error" of the migration in a rotating tank. An additional error occurs because the bubble migrates into a region of different centrifugal acceleration. These errors are a part of the "gravity" migration to be studied. They cannot be eliminated by experimental methods. In contrast, the side migration can be readily detected and eliminated by observing the motion in a plane perpendicular to the plane of rotation. It is the purpose of this study to estimate the magnitude of the migration errors.

The hydrodynamic equations of an inviscid and incompressible fluid in a rotating system are (Squire, reference (12))

$$\nabla \vec{v} = 0$$

(15)

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v}\nabla)\vec{v} + 2[\vec{\omega}\ \vec{v}] + [\vec{\omega}[\vec{\omega}\ \vec{r}]] + \nabla\Omega + \nabla P/\rho = 0,$$

where $\vec{v}$ is the vector of the velocity and $\vec{r}$ that of the coordinates relative to the rotating system. $\Omega$ is the potential of a body force per unit mass, P, the pressure, and $\rho$, the density of the fluid. (15) can be readily deduced from the common hydrodynamic equation and from (4) considering that $\partial\vec{v}/\partial t$ + $(\vec{v}\ \nabla)\vec{v}$ is the acceleration of a fluid particle in the rotating frame. The acceleration in the hydrodynamic equation must be referred to the resting frame. Relation (4) gives the terms which convert the acceleration from one frame to the other.

It is generally assumed that the fluid motion of a migrating bubble is irrotational and that a velocity potential exists. Strictly, this is not true, but it is an acceptable approximation for the major portion of the pulsation. In particular, the motion of a spherical bubble in an inviscid fluid is irrotational in a resting system. In a rotating tank, the fluid is at rest with respect to the moving frame before the initiation of the bubble pulsation. Thus, the motion starts from the rest and for spherical bubbles in a rotating system one may also assume the existence of a velocity potential $\varphi$.

If the flow is axisymmetrical with respect to the x-axis, the Coriolis term in (15) can be written as $-(2\omega\nabla\Psi)/\eta$, where $\Psi$ is Stoke's stream function and $\eta$, the coordinate perpendicular to the axis of symmetry. This form is still not suitable for the Bernoulli equation which is the only possibility to arrive at a solution in a simple way. Hence, we replace this term approximately by $-2\omega\nabla\Psi^*$. The properties of the function $\Psi^*$ will be discussed below. On this basis, (15) takes the form

$$\nabla(-\frac{\partial\varphi}{\partial t} + (\nabla\varphi)^2/2 - 2\omega\Psi^* - \omega^2(x^2 + y^2)/2 + \Omega + P/\rho) = 0. \qquad (15a)$$

Integration yields the Bernoulli equation and, thus, an expression for the pressure P. The components of an external force which may act on the sphere are

$$F_x = 2\pi A^2 \int_{-1}^{+1} P(A) \cos\theta \, d(\cos\theta)$$

$$(16)$$

$$F_y = -2\pi A^2 \int_{-1}^{+1} P(A) \sin\theta \, d(\sin\theta),$$

where A is the radius of the sphere and P(A) the pressure on its surface. Since there are no external forces on a bubble, $F_x$ and $F_y$ must be zero at all times. Evaluation of these integrals will yield the desired expressions for the migration of the bubble.

The velocity potential $\varphi$ of a sphere which has a variable radius A(t) and which moves with the velocities $\Delta y\cdot$ and $\Delta x\cdot$ in the y and x direction

respectively can be represented by that of a variable source and two dipoles. Using polar coordinates r, $\theta$, the potential is

$$\varphi(t, r, \theta) = \frac{A^2 A^{\cdot}}{r} + \frac{A^3 \Delta x^{\cdot}}{2r^2} \cos \theta + \frac{A^3 \Delta y^{\cdot}}{2r^2} \sin \theta. \qquad (17a)$$

For the corresponding function $\Psi^*$, we set tentatively

$$\Psi^*(t, r, \theta) = \Psi^*_s + \frac{A^3 \Delta x^{\cdot}}{2r^2} \sin \theta - \frac{A^3 \Delta y^{\cdot}}{2r^2} \cos \theta. \qquad (17b)$$

Since the source does not contribute to the migration, $\Psi^*_s$ is not needed for our purpose.

The function $\Psi^*$ must have the following properties:

$$\frac{\partial \Psi^*}{\partial y} = \frac{\partial \varphi}{\partial x}$$

$$\qquad (17c)$$

$$\frac{\partial \Psi^*}{\partial x} = - \frac{\partial \varphi}{\partial y}.$$

For a plane flow, these requirements are satisfied by the stream function. For a three dimensional flow, as it is of interest here, a function $\Psi^*$ probably does not exist. In fact, (17a) and (17b) do not satisfy (17c). Still, (17b) is useful as an approximation. It is readily verified that the second terms of (17a) and (17b) satisfy the first row of (17c), but not the second row. Also, the third terms of (17a) and (17b) satisfy the second row of (17c), but not the first one. Comparison with (15) shows that the components of $\Psi^*$ which satisfy (17c) are just those which determine the Coriolis acceleration. Of course, this does not mean that (17b) is anything better than an approximation. When inserted into the integrals (16) those terms of (17b) which contribute at all are correct at the upper and at the lower limit, but faulty at the mid-point of the range of integration. However, at this point the integrant vanishes because of the sin $\theta$ or cos $\theta$ under the integral. Thus, the error is zero at both limits and at the mid-points of the integration. Since the integrants of (16) contribute most strongly near the limits, where $\Psi^*$ is most nearly correct, the use of (17b) is justified as an approximation.

In the x - y coordinate system, used so far, the positive x-axis points away from the center of the rotation. The water in a tank fixed to this axis will be pressed in the positive x-direction and the "gravity" migration will be in the negative x-direction. In underwater explosion work, the z-coordinate commonly refers to the vertical coordinate pointing toward the water surface. The origin of z is at the point of explosion. Denoting the "upward" migration by $\Delta z$, we have

$$x = R - \Delta z + r \cos \theta,$$

where R is the distance between the center of rotation and the point of explosion. The y-coordinate points in the "horizontal" direction, i.e. parallel to the water surface, if its curvature is neglected.

When using the velocity potential (17a) in the Bernoulli equation it must be remembered that the coordinates of $\varphi$ refer to a frame which moves with $\Delta x\cdot$ and $\Delta y\cdot$ with respect to the rotating frame. Since $\partial \varphi / \partial t$ must be referred to a frame which is at rest with respect to the rotating frame, appropriate convective terms must be used to account for the motion of these frames. The Bernouilli equation then takes the form

$$-\frac{P}{\rho} = -\frac{\partial \varphi}{\partial t} + (\nabla \varphi)^2 /2 + \Delta x \cdot \frac{\partial \varphi}{\partial x} + \Delta y \cdot \frac{\partial \varphi}{\partial y}$$

$$-\frac{2\omega A^3}{2r^2} (\Delta x \cdot \sin \theta - \Delta y \cdot \cos \theta) - 2\omega \psi_s^*$$

$$-\frac{\omega^2}{2}\left[ (R - \Delta z)^2 + 2(R - \Delta z) r \cos \theta + r^2 \cos^2 \theta \right.$$

$$\left. + \Delta y^2 + 2\Delta y\, r \sin \theta + r^2 \sin^2 \theta \right]$$

$$- g_x\, r \cos \theta - g_y\, r \sin \theta.$$

(18a)

The potential $\Omega$ and the "gravitational" components $g_x$ and $g_y$ do not refer to the gravity of the earth. Above, the body force per unit mass in a rotating tank has been loosely called "gravity", because it is the purpose of the rotating tank to simulate gravity. $g_x$ and $g_y$ are used in this sense. Actual

high gravity tanks are built with a vertical axis of rotation, so that $g_x$ and $g_y$ due to the true gravity are zero. The purpose of introducing $\Omega$ was to permit an alternative discussion of the effect of rotation. Thus, in the above and the following relations either $\omega$ or $g_x$ and $g_y$ must be zero.

Insertion of (18a) into (16) yields the following simultaneous differential equations for the bubble migration in a rotating tank when $\Delta x$ is replaced by $-\Delta z$:

$$\Delta y^{\cdot\cdot} + 3 \frac{A^{\cdot}}{A} \Delta y^{\cdot} - 2\omega\Delta z^{\cdot} + 2\omega^2\Delta y + 2g_y = 0$$

$$\Delta z^{\cdot\cdot} + 3 \frac{A^{\cdot}}{A} \Delta z^{\cdot} \pm 2\omega\Delta y^{\cdot} - 2\omega^2(R - \Delta z) - 2g_x = 0$$

(18b)

The terms with $2\omega$ comprise the Coriolis migration, those with $\omega^2$ the migration caused by the centrifugal acceleration, then $g_x = g_y = 0$.

Setting $\omega = 0$, one may use $g_x$ and $g_y$ in connection with the relationship for the acceleration in rotating systems (4) to check this result. Assume for a moment that $g_x$ and $g_y$ are the components of gravity, i.e. assume a resting system with the x-axis not exactly vertical. Both $g_x$ and $g_y$ slant downward and migration by buoyancy is vertically upward. Thus, $\Delta x^{\cdot}$ and $\Delta y^{\cdot}$ are in the opposite direction of $g_x$ and $g_y$ respectively. This is in agreement with (18b) since $\Delta x = -\Delta z$. For the migration in a rotating system $g_y$ and $g_z$ in (18b) cannot be directly replaced by the centrifugal and Coriolis accelerations (4). Only the terms $x\omega^2$ and $y\omega^2$ establish a "gravitational field" in the rotating system which causes buoyancy. The Coriolis terms do not produce such effects, but are merely a device to describe the motion of a body in a rotating system employing the concept that the body behaves as if it were subjected to additional forces, the Coriolis forces. To obtain the mass of the body which is of our interest, namely the pulsating bubble, we write (18b) in the form

$$\frac{d}{dt} \left\{ \frac{1}{2} \frac{4\pi A^3}{3} \Delta y^{\cdot} \right\} + g_y \rho \frac{4\pi A^3}{3} = 0 .$$

(18c)

This equation can be interpreted as an equation of motion of a body which has the mass $\frac{1}{2} \rho \frac{4\pi A^3}{3}$ and is subjected to the force of buoyancy $g_y \rho \frac{4\pi A^3}{3}$.

This reflects the well known situation that the virtual mass of a spherical bubble in translatory motion is one-half of the mass of the water displaced by it. The Coriolis force which causes the side motion of the bubble is equal to $2\omega\Delta z^{\cdot} \pi 1/2 \rho \frac{4\pi A^3}{3}$. Under the influence of this term the bubble moves

in the positive y-direction. A direct replacement of $g_y$ by $y^{\cdot\cdot}$ according to (4) would have resulted in a Coriolis migration in the negative y-direction and it would be twice as strong. The above consideration of the Coriolis force yields the same result as that obtained by means of the approximate function $\Psi*$.

## V. GRAVITY MIGRATION

As a first step, we calculate the upward migration in a resting system due to a constant acceleration g. For this case, the differential equation (18b) has the solution

$$\Delta Z = 2 g \int_0^t \frac{dt}{A^3} \int_0^t A^3 \, dt \tag{19}$$

which has been found independently by Herring 10/ and Zoller 8/. To evaluate this expression, we need a relationship for the bubble radius A as a function of time. The strict relationship is exceedingly complicated, but according to a proposal of Zoller a satisfactory approximation can be obtained by

$$A^3(t) = A_m^{\ 3} \left(1 + c - c \left(\frac{2t}{T} - 1\right)^2\right) = A_m^{\ 3} c \left(\alpha^2 - \tau^2\right) . \tag{20}$$

Here, $A_m$ is the minimum bubble radius, T, the period of pulsation, $\tau$, the reduced time, and

$$1 + c = A_M^{\ 3}/A_m^{\ 3}$$
$$\alpha^2 = 1 + c^{-1}.$$

Since the maximum radius $A_M$ is commonly substantially larger than the minimum radius $A_m$, $c \gg 1$ and $\alpha \sim 1$. At every place, where it is permissible, we will neglect $c^{-1}$ in comparison with unity, in particular we can set $1 + \alpha \sim 2$, but $\alpha - 1 \sim 1/2 c$. However, these approximations cannot be made a priori, and each case must be considered separately before the simplification is introduced. In order to avoid the presentation of overly complicated equations only the final simplified results are given below.

Integrating (19) once we obtain for the velocity of migration

$$\Delta Z^{\cdot} = g T \frac{1}{a^3} \int_{-1}^{\tau} a^3 \, d\tau = \frac{gT}{3} \left(\tau + \frac{2}{\alpha - \tau}\right) , \tag{21}$$

where

$$a = A/A_m.$$

The migration is found to be

$$\Delta Z(\tau) = \frac{g\,T^2}{2} \int_{-1}^{\tau} \frac{d\tau}{a^3} \int_{-1}^{\tau} a^3\,d\tau \tag{22}$$

$$= \frac{g\,T^2}{6} \left[ \frac{\tau^2 - 1}{2} + 2 \ln \frac{\alpha + 1}{\alpha - \tau} \right].$$

This relation holds for first cycle of the bubble pulsation; we denote the bubble period by $T_1$ and the value of $\alpha$ appropriate to this cycle by $\alpha_1$. At the bubble maximum $\tau = 0$, thus the migration from the moment of the explosion up to the first bubble maximum is

$$\Delta Z_{1M} = \frac{g\,T_1^2}{6} \left[ 2 \ln \frac{\alpha_1 + 1}{\alpha_1} - \frac{1}{2} \right]. \tag{23}$$

Here, $c^{-1}$ can be neglected in comparison with unity and we obtain with $\alpha_1 = 1$

$$\Delta Z_{1M} = \frac{g\,T_1^2}{6} \left[ 2 \ln 2 - \frac{1}{2} \right] = \frac{g\,T_1^2}{8} \; 1.5816. \tag{24}$$

The term $gT^2/8$ is the distance a free body would have moved under gravity at the time $T/2$, starting from rest. Thus, the average acceleration of the migrating bubble is 1.5816 times that of gravity. A non-pulsating gas filled balloon rises in the water with an acceleration of 2g, as shown in equation (18b). Thus, the expanding bubble rises more slowly than the stationary spherical bubble.

Since the bubble remains spherical during the expansion, (24) should be a good approximation for the migration during this phase.

The migration from the moment of the explosion up to the first bubble minimum is

$$\Delta Z_{1m} = \Delta Z_1 = \frac{g\,T_1^2}{3} \ln 4 \; c_1. \tag{25}$$

This relation affords a possibility to study the dependence on migration of $c_1$ or $A_M/A_m$. It is known that a bubble contracts to a smaller size when under the influence of gravity. Thus, $A_M/A_m$ must depend on the Froude number

$$F = A_M/g\,T^2. \tag{26}$$

To evaluate F, we use the following interrelationship between T and $A_M$

$$T = \sqrt{\frac{3}{2}} \frac{\tilde{t}}{\tilde{a}_M} \frac{A_M}{\sqrt{g \, Z_o}} = 1.96 \frac{A_M}{\sqrt{g \, Z_o}} . \tag{27}$$

Here $\tilde{t}$ is the dimensionless period of the classic bubble theory and $\tilde{a}_M$, the dimensionless maximum radius, 11/. $Z_o$ is the total hydrostatic head at the point of explosion, namely depth + 34 ft (fresh water). The numerical value $\tilde{t}/\tilde{a}_M = 1.6$ used in (27) is appropriate for a variety of explosives and firing conditions. The Froude number then becomes

$$F = \frac{1}{3.84} \frac{Z_o}{A_M} \tag{28}$$

and (25)

$$\frac{\Delta Z_1}{Z_o} = \frac{3.84}{3} \left( \frac{A_M}{Z_o} \right)^2 \ln 4c_1 = \ln 4c_1 / 11.5 F^2 . \tag{29}$$

Commonly, bubble migration is calculated by the semi-empirical formula

$$\Delta Z = CW^{1/2} / Z_o . \tag{30}$$

The constant C depends on the property of the explosive, e.g. for TNT C = 80. The equivalent alternative form

$$\frac{\Delta Z_1}{Z_o} = 1.75 \left( \frac{A_M}{Z_o} \right)^{3/2} = 0.233 \, F^{-3/2} \tag{31}$$

is more general and holds for most explosives, 7/. Comparison of (29) and (31) yields

$$\ln 4c_1 = 1.37 \left( \frac{A_M}{Z_o} \right)^{-1/2} = 2.68 \, F^{1/2} . \tag{32}$$

For $Z_o/A_M = 5$, the above relation yields $c_1 = 5.4$ and $A_M/A_m = 1.86$. Then, $\alpha_1 = 1.09$. For a non-migrating bubble (which is not covered by (32)), $A_M^1/A_m \sim 20$. Since the maximum radius is unaffected, this example illustrates

the drastic change of the minimum bubble radius caused by migration. It is also seen that the approximation $\alpha = 1$ is barely acceptable for the above conditions.

The rate of rise of the bubble in the second cycle is

$$\Delta Z^{\bullet}{}_2 (\tau) = \frac{g \, T_2}{3} \left[ \tau + \frac{4}{\alpha_2 - \tau} + \frac{2}{\alpha_2 + \tau} \right] \quad (33)$$

and the migration

$$\Delta Z_2 (\tau) \approx \frac{g \, T_2^2}{6} \left[ \frac{\tau^2 - 1}{2} + 4 \ln \frac{\alpha_2 + 1}{\alpha_2 - \tau} + 2 \ln \frac{\alpha_2 + \tau}{\alpha_2 - 1} \right]. \quad (34)$$

Here, $T_2$ is the period of the second cycle and $\alpha_2$ the appropriate value of $\alpha$ for this cycle.

At the end of the first cycle and the beginning of the second cycle the rates of rise $\Delta Z^{\bullet}_{1m}$ and $\Delta Z^{\bullet}_2$ ($\tau = - 1$) coincide only if $T_1 = T_2$ and $\alpha_1 = \alpha_2$. For undamped pulsations of non-migrating bubbles these magnitudes are equal. Energy disipation near the bubble minimum as well as upward migration change these magnitudes, so that according to our analysis the rate of migration changes discontinuously at the bubble minimum. It is possible to remove this discontinuity by a proper choice of the integration constant:

$$\Delta Z^{\bullet}_2 (\tau) = \frac{g \, T_2}{3} \left[ \tau + \frac{2}{\alpha_2 - \tau} + \frac{4}{\alpha_1^2 - 1} \frac{T_1}{T_2} \frac{\alpha_2^2 - 1}{\alpha_2^2 - \tau^2} \right] \quad (33a)$$

$$\Delta Z_2 (\tau) = \frac{g \, T_2^2}{6} \left[ \frac{\tau^2 - 1}{2} + 2 \left\{ 1 + \frac{\alpha_2^2 - 1}{\alpha_1^2 - 1} \frac{T_1}{T_2} \right\} \ln \frac{\alpha_2 + 1}{\alpha_2 - \tau} + 2 \frac{\alpha_2^2 - 1}{\alpha_1^2 - 1} \frac{T_1}{T_2} \ln \frac{\alpha_2 + \tau}{\alpha_2 - 1} \right]. \quad (34a)$$

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For the purposes of this study the simpler forms given above are sufficiently accurate and the total migration up to the second bubble maximum becomes

$$\Delta Z_{2M} + \Delta Z_1 = \frac{g\, T_2^2}{6}\left[4 \ln 2 - \frac{1}{2} + 2 \ln 2c_2\right] + \frac{g\, T_1^2}{6}\, 2 \ln 4c_1 \tag{35}$$

and up to the second bubble minimum:

$$\Delta Z_2 + \Delta Z_1 = gT_2^2 \ln 4c_2 + \frac{g\, T_1^2}{6}\, 2 \ln 4c_1 \;. \tag{36}$$

If the period T and the coefficient c were the same in the first and second cycle, the second cycle migration would be three times that of the first cycle.

## VI. SIDEWARD MIGRATION IN A ROTATING TANK

We designate the sideward coordinate in a rotating tank by y and retain z for the upward coordinate. The approximate equation for the bubble motion is according to the considerations of Part IV of this report

$$\Delta y^{\cdot\cdot} + 3\frac{a^{\cdot}}{a}\Delta y^{\cdot} - 2\,\omega\,\Delta z^{\cdot} + 2\,\Delta y\omega^2 = 0 \;. \tag{37}$$

This equation can be readily evaluated following the previous lines if the last term is neglected and $\Delta z^{\cdot}$ is assumed to result from a constant acceleration $R\omega^2$, where R is the distance between the point of explosion and the center of the rotation. An estimate of the error of this approximation will follow.

The rate of sideward motion is, using (21)

$$\Delta y_i^{\cdot} = \frac{4\,\omega^3\, R}{A^3}\int_0^t\int_0^t A^3\, dt\, dt = \frac{\omega^3\, R\, T_i^2}{a^3}\int_{-1}^{\tau}\int_{-1}^{\tau} a^3\, d\tau\, d\tau \tag{38}$$

$$= \omega^3 R\, T_i^2\left(\frac{\tau^2 + 1}{12} - \frac{1}{2} + \frac{K_1 + K_2}{\alpha_i - \tau} + \frac{K_2}{\alpha_i + \tau}\right),$$

where $K_1$ is the integration constant of the first integration and $K_2$, that

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of the second integration. $K_1$ is readily found to be

$$K_1 = 2/3 \qquad \text{First cycle}$$

$$K_1 = 2 \qquad \text{Second cycle .} \tag{38a}$$

The integration constant $K_2$ is determined so that, at the beginning of the second cycle, $\triangle y\cdot$ has the same value as at the end of the first one, if T and $\alpha$ are equal. (The more elaborate treatment shown before, (33a) and (34a), does not appear to be worthwhile here.) One obtains

$$K_2 = 0 \qquad \text{First cycle}$$

$$K_2 = \frac{2}{3} - \frac{4}{3}(\alpha_1 - 1) \sim \frac{2}{3} . \qquad \text{Second cycle} \tag{38b}$$

The sideward migration becomes

$$\triangle y_1 (\tau) = \frac{\omega^3 R T_1^3}{2}\left(\frac{\tau^3 + 1}{36} - \frac{5\tau + 5}{12} + \frac{2}{3} \ln \frac{\alpha_1 + 1}{\alpha_1 - \tau}\right), \text{First cycle}$$

$$\triangle y_2 (\tau) = \frac{\omega^3 R T_2^3}{2}\left(\frac{\tau^3 + 1}{36} - \frac{5\tau + 5}{12} - \frac{8}{3} \ln \frac{\alpha_2 + 1}{\alpha_2 - \tau}\right.$$
$$\left. + \frac{2}{3} \ln \frac{\alpha_2 + \tau}{\alpha_2 - 1}\right) . \qquad \text{Second cycle} \tag{39}$$

For the sideward migration within the first and second cycle respectively, we find

$$\triangle y_1 = \frac{\omega^3 R T_1^3}{2}\left[-\frac{2}{3}\ln 4c_1 - \frac{7}{9}\right]$$

$$= \omega T_1 \triangle z_1 - \frac{7}{18} \omega^3 R T_1^3 , \tag{40}$$

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$$\Delta y_2 = \frac{\omega^3 R T_2^{\,3}}{2} \left[ \frac{10}{3} \ln 4c_2 - \frac{7}{9} \right]$$

(41)

$$= \frac{5}{3} \omega T_2 \Delta z_2 - \frac{7}{18} \omega^3 R T_2^3 \ .$$

To estimate the error of neglecting $\Delta y$ in (37), we find for **the instant of the** first bubble minimum

$$2\omega \Delta \dot{z}_1 + 2\omega^2 \Delta y_1 = \frac{2\omega^3 R T_1}{3} \left[ 1 + 4c_1 + \omega^2 T_1^2 (\ln 4c_1 - 7/6) \right]$$

(42)

and for the second bubble minimum

$$2\omega \Delta \dot{z}_2 + 2\omega^2 \Delta y_2 = \frac{2\omega^3 R T_2}{3} \left[ 2 + 8c_2 + \omega^2 T_2^2 (5 \ln 4c_2 - 7/6) \right] \ .$$

(43)

In both cases the last term in the braces is negligibly small compared with the terms $4c_1$ or $8c_2$, not only because $c \gg 1$, as generally assumed in our calculations, but also because $\omega^2 T^2$ is a small magnitude in all practical cases. Since the last terms represent the magnitude neglected in (37), the approximation appears to be permissible. The same conclusion is reached, if this comparison is made for other moments of time. The justification of the second approximation which assumes a constant acceleration $\omega^2 R$ in the calculation of $\Delta y$ will become apparent in Part VII, where it will be shown that the change of $\Delta z$ due to the charge of acceleration is exceedingly small. The same result would be obtained for $\Delta y$.

## VII. THE DISTORTION OF THE UPWARD MIGRATION IN A ROTATING TEST TANK

According to the discussion in Part IV, we obtain for the migration in the z-direction (toward the water surface and the center of rotation)

$$\Sigma \Delta z(t) = 2R\omega^2 \int_o^t A^{-3} \int_o^t A^3 \, dt \, dt - 2\omega \int_o^t A^{-3} \int_o^t A^3 \Delta \dot{y} \, dt \, dt$$
$$- 2\omega^2 \int_o^t A^{-3} \int_o^t A^3 \Delta z \, dt \, dt \ .$$

(44)

The first term corresponds to the gravity migration discussed in Section V. The second term is a result of the Coriolis acceleration, and the third accounts for the variation of the centrifugal acceleration with depth. We denote the contribution of the Coriolis term by the subscript c and obtain with (38):

First cycle

$$- \Delta z_{1c}(\tau) = \frac{\omega^4 R T_1^4}{2} \left[ \frac{\tau^4 - 1}{240} - \frac{3(\tau^2 - 1)}{40} - \frac{\tau + 1}{3} + \frac{2}{5} \ln \frac{\alpha_1 + 1}{\alpha_1 - \tau} \right]$$

(45)

Second cycle

$$- \Delta z_{2c}(\tau) = \frac{\omega^4 R T_2^4}{2} \left[ \frac{\tau^4 - 1}{240} - \frac{3(\tau^2 - 1)}{40} - (\tau + 1) + \frac{52}{15} \ln \frac{\alpha_2 + 1}{\alpha_2 - \tau} \right.$$
$$\left. + \left( \frac{2}{5} - \frac{4}{3}(\alpha_2 - 1) \right) \ln \frac{\alpha_2 + \tau}{\alpha_2 - 1} \right].$$

The term $4(\alpha_2 - 1)/3$ in the last expression stems from the integration constant which assures equality of the rate of rise at the end of the first and the beginning of the second bubble cycle. This term is shown for completeness, but will be neglected hence forth. The Coriolis contributions up to the first and second minimum respectively are

$$- \Delta z_{1c} = \frac{\omega^4 R T_1^4}{2} \left[ \frac{2}{5} \ln 4c_1 - \frac{2}{3} \right] = \frac{3\omega^2 T_1^2}{5} \Delta z_1 - \frac{\omega^4 R T_1^4}{3},$$

(46)

$$- \Delta z_{2c} = \frac{\omega^4 R T_2^4}{2} \left[ \frac{58}{15} \ln 4c_2 - 2 \right] = \frac{29\omega^2 T_2^2}{15} \Delta z_2 - \omega^4 R T_2^4.$$

Up to this point, the integrations needed for the various migration terms have been simple. But, the calculation of the last term of the upward migration turned out to be more complicated and involves functions which are not readily available. Therefore, a further simplification is made. For use in (44) $\Delta z$ is approximated by

$$\Delta z_i(\tau) = \frac{\omega^2 R T_i^2}{6} \left[ \frac{\tau - 1}{2} + q_1 + q_2 \tau + q_3 \tau^2 \right],$$

(47)

where  First cycle  Second cycle

$$q_1 = 2 \ln 2 \qquad\qquad q_1 = 2 \ln 2 + 2 \ln 4c_2$$

$$q_2 = \ln 4c_1 \qquad\qquad q_2 = 3 \ln 4c_2$$

$$q_3 = \ln 4c_1 - 2 \ln 2 \qquad\qquad q_3 = \ln 4c_2 - 2 \ln 2 \ .$$

The approximation (47) coincides with (22) and (34) at $\tau = 1$, 0, and -1. Although the nature of the functions and their derivatives differ, the approximation (47) will suffice for the rough estimates which are the purpose of this study.

The migration due to the change of the centrifugal acceleration then becomes for the i-th cycle

$$
\begin{aligned}
- \Delta z_{iz} &= \frac{\omega^2 T_i^2}{2} \int_{-1}^{\tau} \frac{1}{a^3} \int_{-1}^{\tau} a^3 \, \Delta z \, d\tau \, d\tau \\
&= \frac{\omega^4 R T_i^4}{12} \left[ (B_0 + K) \ln \frac{\alpha_i + 1}{\alpha_i - \tau} + K \ln \frac{\alpha_i + \tau}{\alpha_i - 1} + B_1 (\tau + 1) \right. \\
&\qquad \left. + B_2 (\tau^2 - 1) + B_3 (\tau^3 + 1) + B_4 (\tau^4 - 1) \right]
\end{aligned}
\tag{48}
$$

with

$$B_0 = (10q_1 + 2q_3 - 4)/15 \tag{49a}$$

$$= (16 \ln 2 - 4 + 2 \ln 4c_1)/15 \qquad \text{First cycle}$$

$$= (16 \ln 2 - 4 + 22 \ln 4c_2)/15 \qquad \text{Second cycle}$$

$$B_1 = - q_2/4 \tag{49b}$$

$$= - \frac{1}{4} \ln 4c_1 \qquad \text{First cycle}$$

$$= - \frac{3}{4} \ln 4c_2 \qquad \text{Second cycle}$$

$$B_2 = (10q_1 - 4q_3 - 7)/60 \qquad (49c)$$

$$= (28 \ln 2 - 7 - 4 \ln 4c_1)/60 \qquad \text{First cycle}$$

$$= (28 \ln 2 - 7 + 16 \ln 4c_2)/60 \qquad \text{Second cycle}$$

$$B_3 = {}^+ q_2/12 \qquad (49d)$$

$$= \frac{1}{12} \ln 4c_1 \qquad \text{First cycle}$$

$$= \frac{1}{4} \ln 4c_2 \qquad \text{Second cycle}$$

$$B_4 = (2q_3 + 1)/40 \qquad (49e)$$

$$= (2 \ln 4c_i - 4 \ln 2 + 1)/40 \qquad \text{Both cycles}$$

$$K = 0 \qquad \text{First cycle}$$
$$\qquad (49f)$$
$$K = (16 \ln 2 - 4 + 2 \ln 4c_1)/15 \qquad \text{Second cycle}.$$

Because of the non-constancy of the acceleration in a rotating tank, the "upward" migration is changed within the i-th cycle by

$$- \Delta z_{iz} = \frac{\omega^4 R T_i^4}{12} \left[ (B_0 + 2K) \ln 4c_i + 2 (B_1 + B_3) \right]. \qquad (50)$$

In the first cycle

$$- \Delta z_{1z} = \frac{\omega^4 R T_1^4}{12} \ln 4c_1 \left[ 0.139 + \frac{2}{15} \ln 4c_1 \right] \qquad (50a)$$

$$= 0.0348\omega^2 T_1^2 \Delta z_1 + \frac{\Delta z_1^2}{10 R}.$$

and in the second cycle, if the $c_1$ occurring in K is approximately substituted by $c_2$:

$$- \Delta z_{2z} = \frac{\omega^4 R T_2^4}{12} \ln 4c_2 \left[ 0.4176 + \frac{26}{15} \ln 4c_2 \right] \qquad (50b)$$

$$= 0.0348 \omega^2 T_2^2 \Delta z_2 + \frac{13}{90} \frac{\Delta z_2^2}{R}.$$

The total migration in the rotating tank is

$$\Sigma \Delta z_i = \Delta z_i + \Delta z_{ic} + \Delta z_{iz}. \tag{51}$$

## VIII. DISCUSSION OF THEORETICAL AND EXPERIMENTAL RESULTS

A few explosion tests made with the NOL centrifuge made the observation of the sideward migration possible 9/. Unfortunately, an accurate evaluation of this migration proved to be difficult, so that the values given below are rather uncertain. Nevertheless, these tests may serve as an illustration of the various migration components.

The test conditions of one of these tests were as follows:

$$R = 10 \text{ ft}$$

$$\omega = 14.2 \text{ sec}^{-1}.$$

Fig. 4 is a contour-time plot of the bubble showing the upward and sideward migrations. The curves give the two extremes of the bubble dimension in horizontal and vertical extension as a function of time. The bubble center, assumed to be halfway between these extremes, is shown as a dotted curve.

The graph shows a difficulty which is typical of migrating bubble problems, namely the uncertainty of the time of the bubble minimum. The maximum contraction in vertical direction occurs at a different time (4.85 msec) than that in the horizontal direction (5.25 msec). The first instant corresponds to the moment where the lower and upper bubble interface impinge on each other. In pressure records, this instant is manifested by a short, but strong pressure pulse originated by the water hammer. This "spike" is superposed on the bubble pulse originated by the compressed gases in the bubble and it occurs before the pressure maximum of the bubble pulse, Fig. 5. The latter moment corresponds to that of the bubble minimum and it is not necessarily identical with the moment of the maximum lateral contraction. This is nicely demonstrated by the dotted curve representing the upward migration in Fig. 4: Significant upward migration (i.e. regions where the rate of migration is high) begins at $t = 4.85$ msec (vertical minimum of bubble) and seems to end at $t = 5.25$ msec (horizontal minimum). Since significant migration takes place before as well as after the moment of the bubble minimum, the minimum must occur between $t = 4.85$ msec and $t = 5.2$ msec. We have chosen $t = 5.1$ msec and obtain:

| Observed Bubble Periods | Total "Upward" Migration |
|---|---|
| $T_1 = 5.1$ msec | $\Sigma \Delta z_1 = 0.65$ inch |
| $T_2 = 5.2$ msec | $\Sigma \Delta z_1 + \Sigma \Delta z_2 = 1.60$ inch |

Since the difference between $\Sigma \Delta z_i$ and $\Delta z_i$ is small, we set approximately

$$\Delta z_1 = 0.65 \text{ inch}$$

$$\Delta z_2 = 0.95 \text{ inch}.$$

From (40) and (41), we find for the sideward motion

$$\Delta y_1 = 0.03 \text{ inch}$$

$$\Delta y_2 = 0.098 \text{ inch}.$$

The smallness is these magnitudes shows that a quantitative comparison with measured data is a doubtful undertaking, since 3/100 or even 1/10 of an inch are not readily evaluated from a high-speed movie film. Nevertheless, such an evaluation is attempted in Fig. 6 which shows an acceptable agreement between measured and calculated side migration. At the end of the second cycle the calculated side migration is about 30% higher than the experimental one, 0.1 vrs. 0.13 inch. This would be in line with other theoretical results based on spherical bubble shapes. Because of possible baseline shifts, the side migration at the end of the first cycle is uncertain. However, this comparison indicates that our calculations give a satisfactory estimate of the sideward migration.

The Coriolis correction for the "upward" migration as obtained from (46) is:

$$\Delta z_{1c} = -1.5 \cdot 10^{-3} \, \Delta z_1$$

$$\Delta z_{2c} = -7 \cdot 10^{-3} \, \Delta z_2$$

and the correction because the variation of the acceleration is

$$\triangle z_{1z} = -0.6 \cdot 10^{-3} \triangle z_1$$

$$\triangle z_{2z} = -1.3 \cdot 10^{-3} \triangle z_2.$$

It is interesting that the two "errors" have the same sign. However, the error due to change of acceleration is much smaller than that caused by the Coriolis effect. The total error in "upward" migration is 0.8 percent, hence negligibly small for practical purposes where experimental errors of much larger magnitude are to be expected.

Since the conditions of this example are typical of a high gravity tank setup, one is justified in concluding that the errors in the upward migration of explosion bubbles which are introduced by the rotation will not constitute an obstacle against adopting this type of high gravity tank.

Equations (46) and (50) show that the errors in upward migration involve terms like $\omega^2 T^2$, $\omega^4 T^4 R/\triangle z$, etc. With $g_z = R\omega^2$, the first of these terms is $g_z T^2/R$. Hence, in this case the error term is inversely proportional to R, for any given value of $g_z T^2$. As the other terms show, the total error is not necessarily a linear function of $R^{-1}$, but involves higher negative powers of R. This confirms the argument that a slowly rotating centrifuge with a large arm produces less distortion of the bubble migration than a small, fast rotating centrifuge.

---
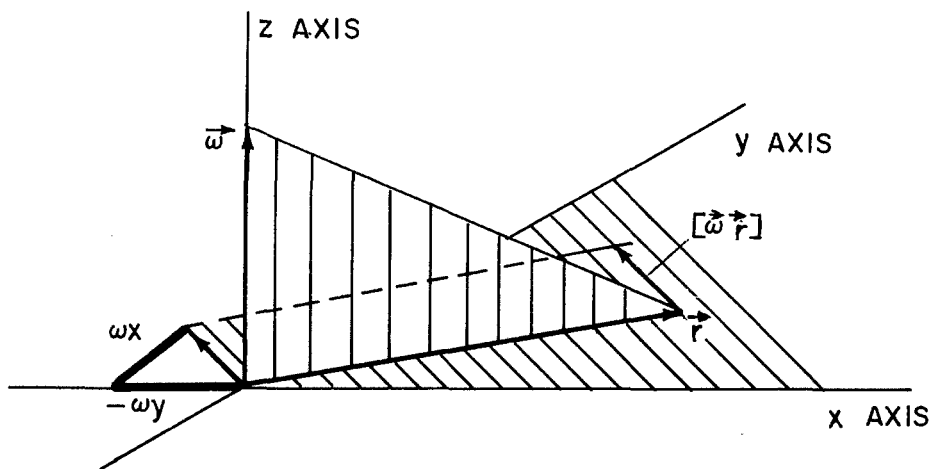
FIG. I  COMPONENTS OF THE VECTOR $[\vec{\omega}\,\vec{r}]$
(THE VECTOR $[\vec{\omega}\,[\vec{\omega}\,\vec{r}]]$ IS NOT SHOWN)
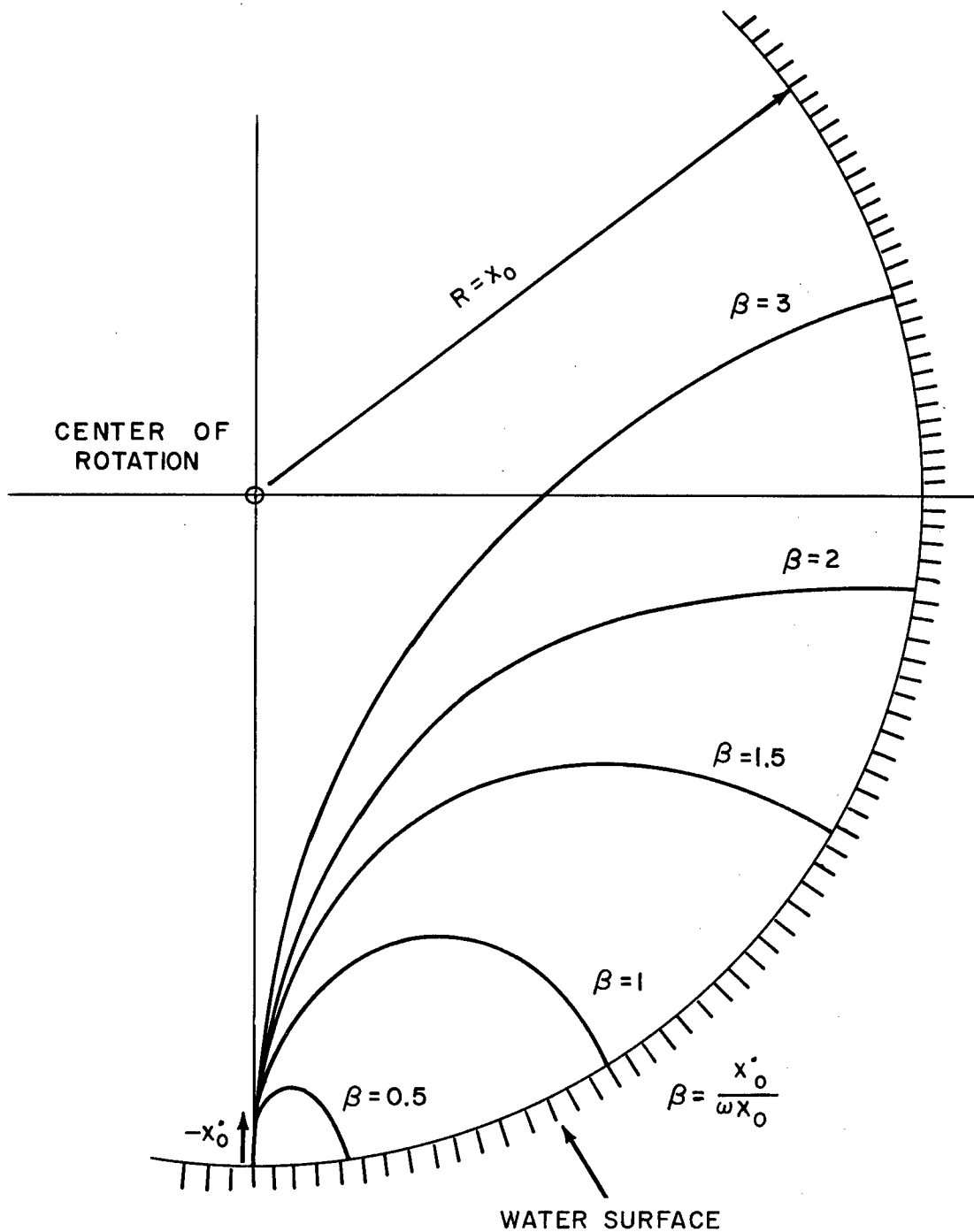
FIG. 2 TRAJECTORIES OF SPRAY PARTICLES WHICH MOVE
INITIALLY TOWARD THE CENTER OF ROTATION, AS
SEEN BY A ROTATING OBSERVER
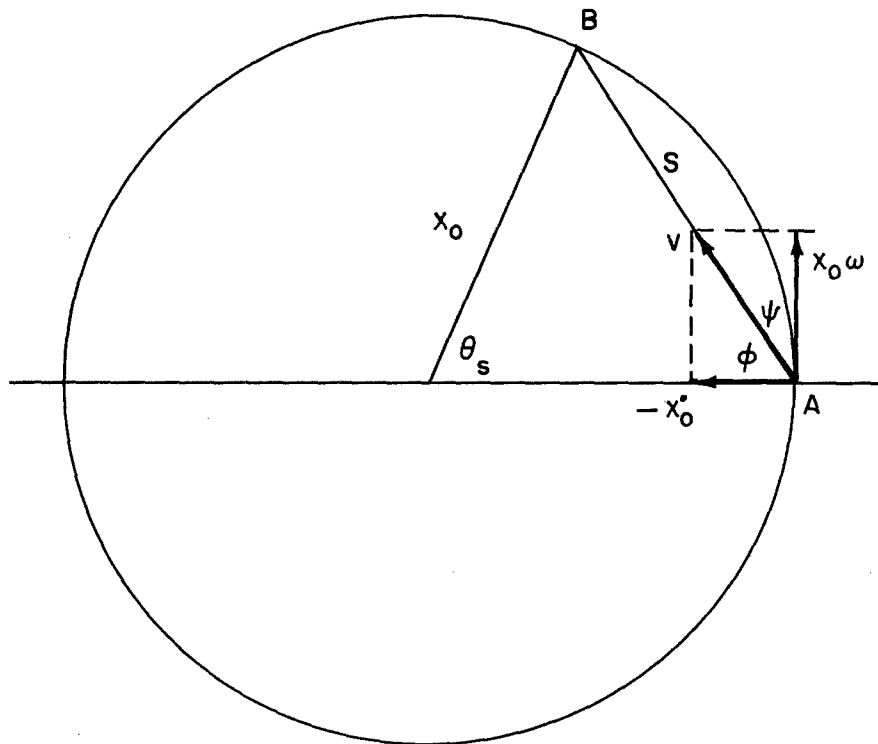
FIG. 3 MOTION OBSERVED BY A RESTING OBSERVER

FIG. 4   BUBBLE CONTOURS AND MIGRATION IN UPWARD AND SIDEWARD DIRECTION

NON-MIGRATING
BUBBLE

t = 0.5 T

t = T

FIG. 5  SHAPE OF THE BUBBLE PULSE. T MEANS THE
DURATION OF THE BUBBLE OSCILLATION.
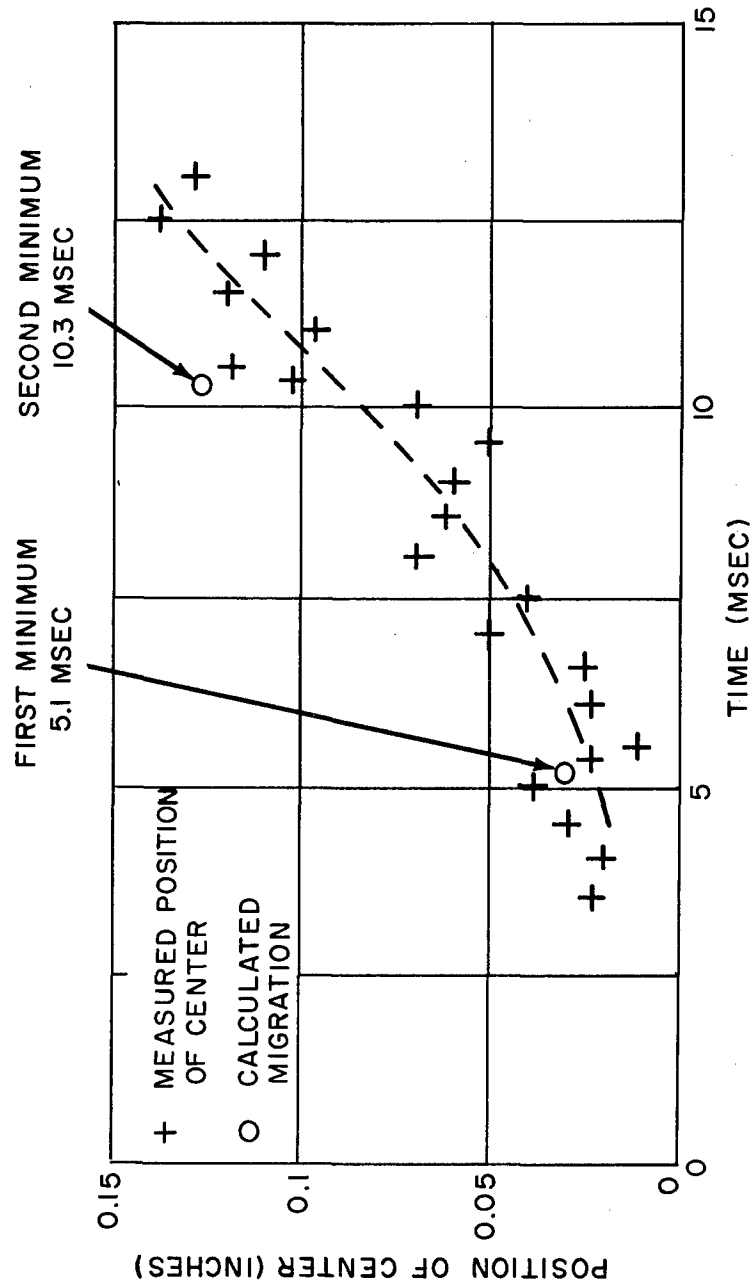
FIG. 6 SIDEWARD MIGRATION

REFERENCES

(1) "Bubble Migration on a Small Scale", H. G. Snay, 4th Conference on Research on Ship Protection Against Underwater Explosions, NavShips 250-423-14, 4-6 Dec 1951, Confidential

(2) "Feasibility Study and Design Investigation for a Scaled Underwater Explosion Test Facility", Washington Technological Associates, Inc., Final Report (Contract NOrd 17605), March 1958, Confidential
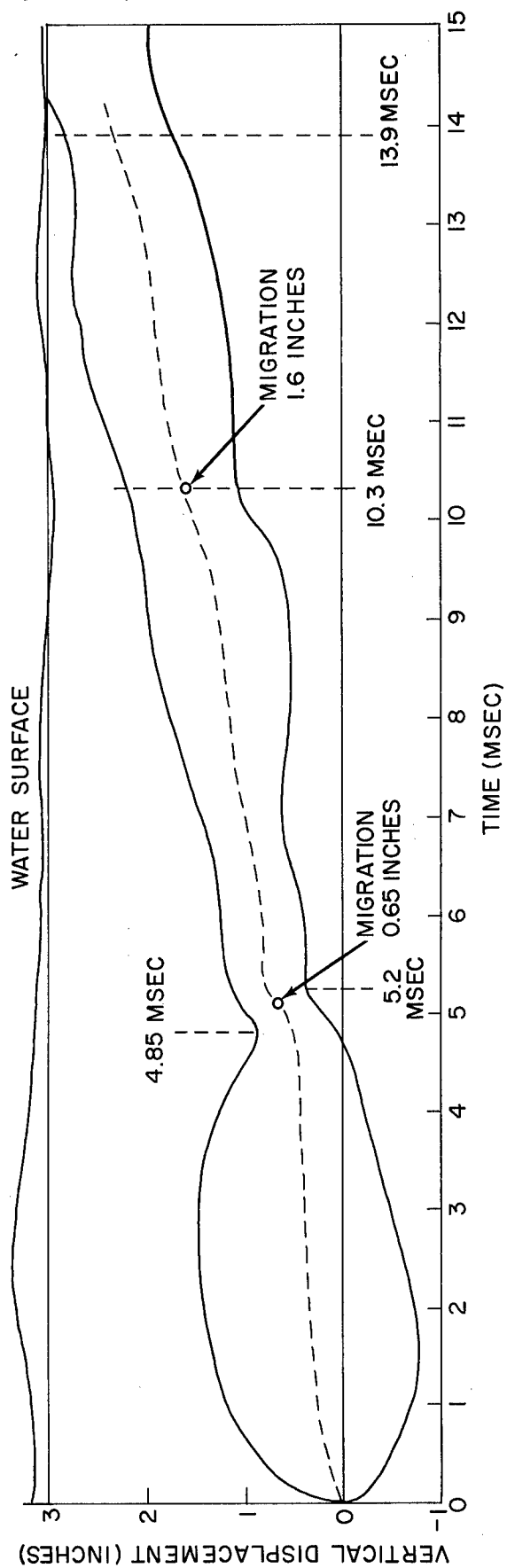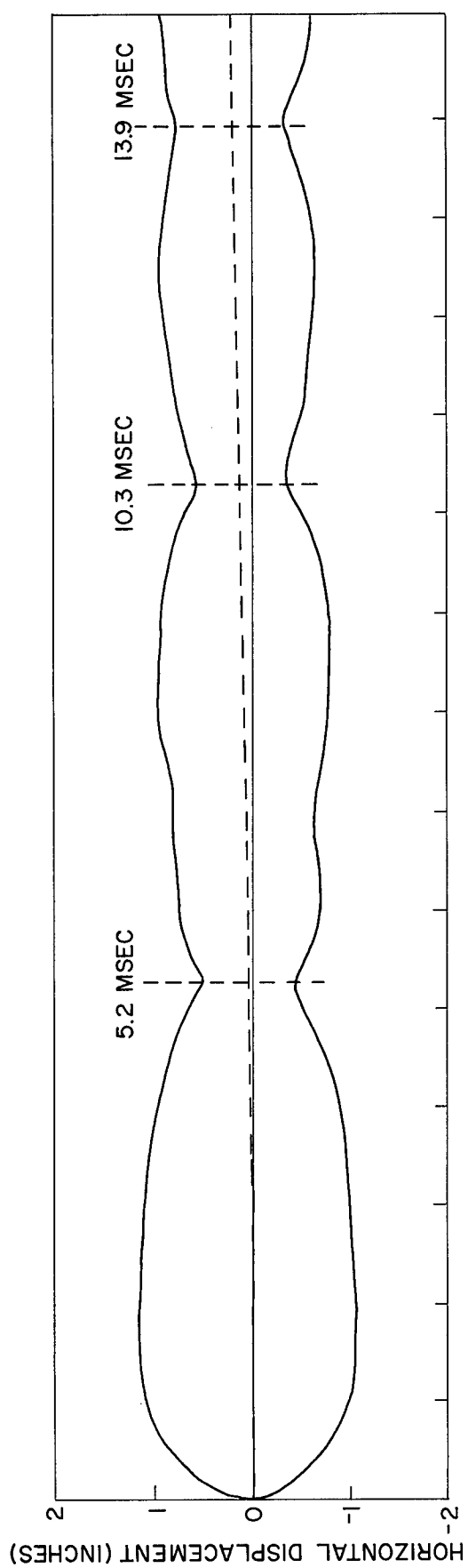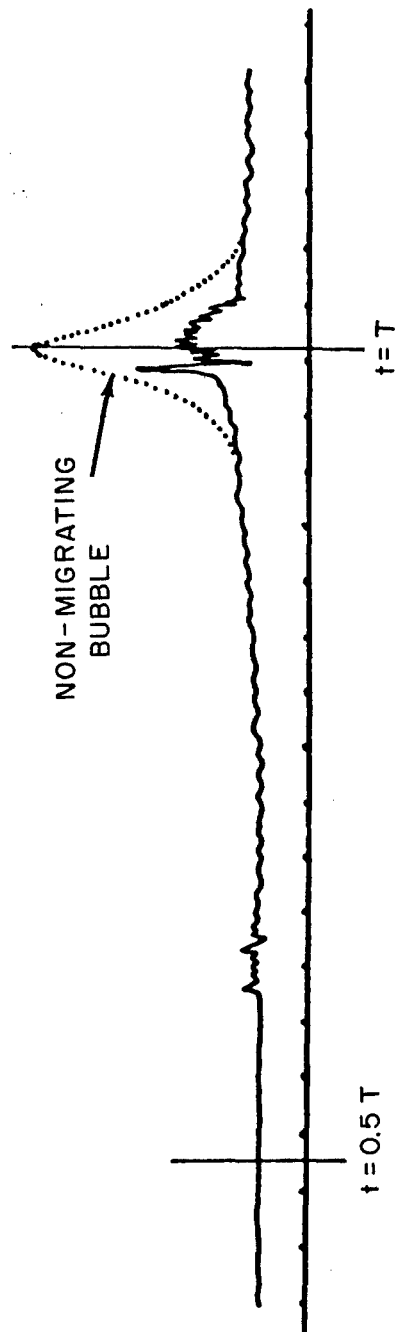
(3) "Scaling Problems in Underwater Ballistics", H. G. Snay, "Selected Topics on Ballistics", Pergamon Press, London, 1959, Unclassified

(4) DTMB memorandum by George Hudson on file at NOL, E Department

(5) "Small Scale Experiments to Determine Migration of Explosion Gas Globes Towards Submarines", H. G. Snay, J. F. Goertner, and R. S. Price, NAVORD Report 2280, 1 July 1952, Confidential

(6) "Vacuum Tank Studies of Gravity Migration of Underwater Explosion Bubbles", John F. Goertner, NAVORD Report 3902, 1 Mar 1956, Confidential

(7) "The Hydrodynamic Background of the Radiological Effects of Underwater Nuclear Explosions", H. G. Snay, NAVWEPS Report 7323, 23 Sept 1960, Confidential

(8) "Aufsteigen der Pulsierenden Schwadenblase", R. Zoller. Forschungsanstalt Graf Zeppelin, Stuttgart, Germany, 1944, Unclassified

(9) "Underwater Explosion Tests in a Preliminary High-Gravity Tank Accelerated by a Centrifuge", R. S. Price, NAVWEPS Report 7365, 15 Jan 1961, Unclassified

(10) "Underwater Explosions", R. H. Cole, Princeton University Press 1948, page 308

(11) "Underwater Explosion Phenomena: The Parameters of a Non-Migrating Bubble...", H. G. Snay and E. A. Christian, NAVORD Report 2937, 1 Feb. 1952, Unclassified.

(12) "Rotating Fluids", H. B. Squire. Surveys in Mechanics. G. I. Taylor 70th Anniversary Volume. Cambridge University Press. 1956, Unclassified.

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## SUBJECT ANALYSIS OF REPORT

| DESCRIPTORS | CODES | DESCRIPTORS | CODES | DESCRIPTORS | CODES |
|---|---|---|---|---|---|
| Underwater | UNDE | Water | WATR | Lateral | LATR |
| Explosions | EXPS | Surface | SURA | Distortion | DSTO |
| Gravity | GRAV | Acceleration | ACCL | Vertical | VERT |
| Tank | TANZ | Centifuge | CENT | Data | DATA |
| High | HIGH | Tank (Design) | TANZD | Vacuum tank | VACT |
| Migration | MIGR | Linear | LINN | | |
| Pressure | PRES | Accelerator | ACCR | | |
| Bubbles | BUBB | Equations | EQUA | | |
| Rotating | ROTA | Mathematics | MATH | | |
| Pulsating | PULS | Motion | MOTI | | |
| Buoyancy | BUOA | System | SYST | | |
| Air | AIRE | Spray | SPRY | | |

Naval Ordnance Laboratory, White Oak, Md.
(NOL technical report 61-145)
MIGRATION OF EXPLOSION BUBBLES IN A ROTATING
TEST TANK, (U), by Hans G. Snay.  Oct. 1961.
23p. illus. diagr.  Task NOL-440.
UNCLASSIFIED

In a rotating tank the migration of explosion
bubbles is distorted by the Coriolis accelera-
tions.  The bubble is deflected to the side and
the "upward" (i.e. toward the water surface)
motion is reduced.  This introduces a system-
atic error which cannot be eliminated by experi-
mental methods.  Calculations show that the
error can be made negligibly small by increas-
ing the length of the arm of the centrifuge.

1. Explosions,
   Underwater
2. Tanks, High
   gravity
3. Bubbles,
   Explosion
4. Bubbles –
   Migration
I. Title
II. Snay, Hans G.
III. Project

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